INVESTIGATION OF THE ENTRANCE LENGTH OF A PLANE JET DEVELOPING IN UNCONFINED AND CONFINED CROSS FLOWS

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The results of a hot wire anemometer study are presented and a method is proposed for calculating the mean characteristics of the entrance section of a plane jet in unconfined and confined cross flows. The calculated and experimental data are compared.

A jet issuing from a plane slit at an angle to the flow develops under nonsymmetrical conditions: on the upstream side it constitutes a curved permeable wall relative to the flow, while on the downstream side there is formed a circulation zone, in which the pressure is considerably lower than the pressure in the free stream, and the velocities are small. The entrance length of such a jet comprises a core of constant total pressure [1] and two differently developing mixing zones — an outer (upstream) and an inner (downstream) zone (see Fig. 1).

By experimentally investigating the entrance length of a plane jet in an unconfined cross flow [1] it was established that the width of the inner mixing zone is approximately the same as in a submerged jet. However, the development of the outer zone is more intense. Judging from the literature available, the fluctuation characteristics of the entrance length of a plane jet in a cross flow have not previously been studied.

Below we present the results of an experimental investigation of the mean and fluctuation characteristics of the flow in the entrance section of a plane turbulent jet developing in unconfined and confined cross flows. We also give the results of calculating the width of the outer and inner mixing zones for a plane jet in a confined cross flow, obtained by extending the solution of [2] for a jet in an unconfined cross flow to the case of development of a jet in a channel. The results of the calculations are compared with the experimental data.

The experiment was carried out on the apparatus described in [3]. The jet was blown through a slit of width $b_0 = 4 \cdot 10^{-3}$ m and length 0.17 m into an unconfined flow and into a flow bounded by channel walls. In this case the nonuniformity of the velocity profile at the nozzle exit did not exceed 2%. In investigating the entrance section in the wind tunnel we installed additional screens and dust filters. As a result, the flow was comparatively clean, and the intensity of turbulence was of the order of 0.35% at velocities up to 30 m/sec.

During the experiment we measured the mean and fluctuation characteristics of the jet by means of an ST-1 hot wire anemometer apparatus designed and built at the Kazakh State University [4]. The hot wire anemometer system employed consists of a hot wire probe, a digital voltmeter, a rms millivoltmeter and a linearizer with a linearization range of 1-100 m/sec, a frequency range of 0-30 kHz, and an error of not more than 1.5%. The maximum of the probe frequency range varies from 150 to 100 kHz, depending on the wire diameter $(3\text{-}7) \cdot 10^{-6}$ m. The platinum wire, 5 μm in diameter, was covered with a copper sheath which was etched away over a length of 1 mm. The velocity vector and its fluctuations were measured using a probe in which the axis of the wire was directed along the nozzle slit. The probe holder was located in the circulation zone. The electrode length was so chosen that the holder did not interfere with the jet as the probe wire passed through it. The measurements were made in sections perpendicular to the trajectory of the jet which was determined approximately from the direction of the velocity vector measured with a two-channel tube (channel diameter $6 \cdot 10^{-4}$ m) near the jet axis at a certain distance from the nozzle under the assumption that on the entrance length the radius of curvature of the axis is constant.

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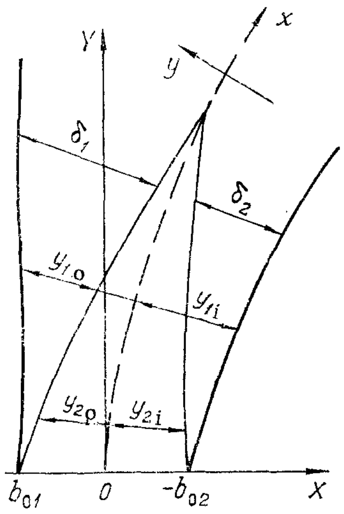


Fig. 1. Diagram of the entrance length of a plane jet in a cross flow.

The entrance length was investigated for a jet in an unconfined flow and in a channel of relative height $H/b_0 = 20$ on the interval of ratios of the dynamic heads of the jet and the flow from 3 to 14. During the measurements the jet temperature differed from the flow temperature by not more than 6°. The probes were calibrated using an air flow at the temperature of the jet.

The Reynolds stresses were measured with a "diagonal" wire mounted at angles of 45° and 135° to the flow [5, 6].

It should be noted that the results of a hot wire probe investigation of the mean and fluctuation characteristics of a submerged jet carried out using the apparatus described above are in good agreement with the data of other authors.

In Fig. 2 we have plotted the profiles of the modulus of the mean velocity vector and the intensity profiles for the turbulent fluctuations of the velocity vector in sections perpendicular to the axis of a jet propagating in a channel at a jet/flow velocity ratio $u_0/V_\infty = 2.24$ and in an unconfined flow at $u_0/V_\infty = 3.34$. Clearly the jet core velocity varies significantly over the radius, increasing toward the trailing edge of the jet. This is associated with the bending of the jet under the influence of the large pressure gradient. It should be noted that investigations in other regimes showed that the variation is the greater the smaller the quantity $u_0/V_\infty$.

Comparison of the velocity profiles for jets in a channel and in an unconfined flow shows that the radial variation of the core velocity is more intense for the jet in the confined flow. This was to be expected since the pressure gradient at the root of the jet is substantiall higher in the channel than in the unconfined flow. Processing of the velocity profiles in the usual dimensionless coordinates showed that they are similar and well-described by the Schlichting formula. It is also clear from the graphs that the intensity of the turbulent fluctuations of the velocity vector is greater in the outer than in the inner mixing zone; with distance from the nozzle exit the difference in intensity decreases but remains appreciable over the entire entrance length. A similar picture is observed in the other regimes investigated for jets in both confined and unconfined flows.

A comparison of the graphs of the intensity of the turbulent fluctuations in jets propagating in unconfined and confined flows shows that the level of the fluctuations in both upstream and downstream mixing zones is higher for the unconfined than for the channel flow. It is also clear from Fig. 2 that the intensity of turbulence in the jet core increases somewhat with distance from the nozzle exit.

In Fig. 3 we have reproduced the results of measurements of the Reynolds stresses in two sections of a jet propagating in a channel. The same figure also shows the experimental points obtained by Wignanski and Fiedler and by Childs in a free mixing layer [7] and the mean velocity profile. Clearly near the nozzle exit the friction stresses in the outer mixing zone are almost twice as great as those in the inner zone. With distance from the nozzle exit the difference between the friction stresses in these zones decreases. In both zones the friction stresses increase with distance from the nozzle exit, but the rate of increase is greater in the inner zone. The growth of the friction stresses in the outer zone is evid-

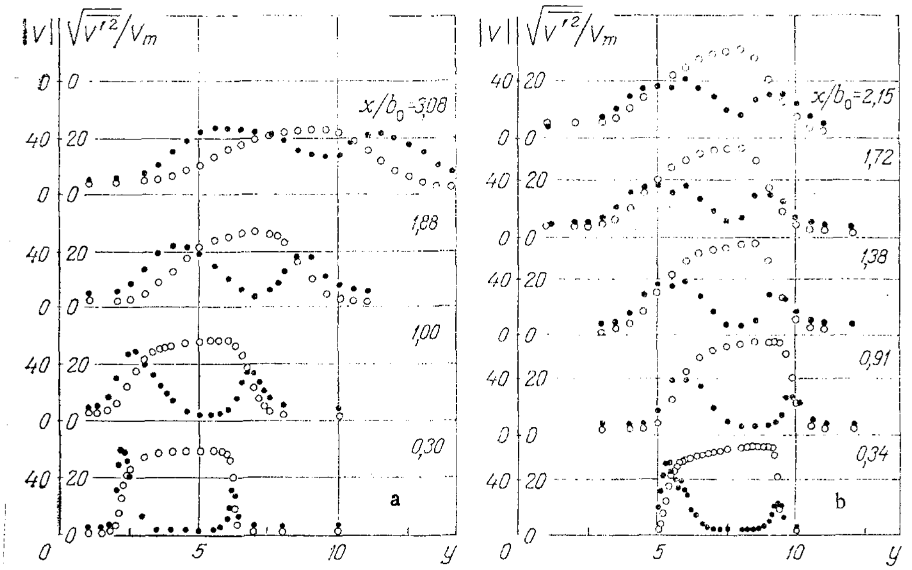


Fig. 2. Profiles of the velocity vector (open circles) and of the intensity of the turbulent fluctuations of the velocity vector (black symbols) in cross sections of the entrance length of the jet: a) $H/b_0 = \infty$, $u_0/V_\infty = 3.34$; b) $H/b_0 = 20$, $u_0/V_\infty = 2.24$; y, mm.

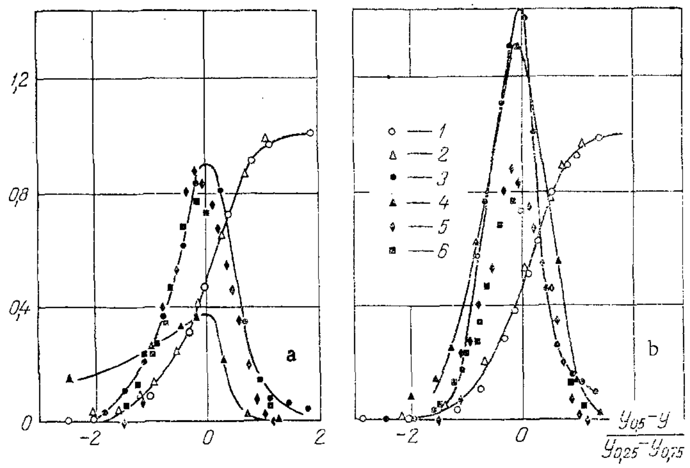


Fig. 3. Turbulent shear stresses in upstream and downstream mixing zones: a) $x/b_0 = 0.325$; b) $x/b_0 = 0.8$; 1) $(u - u\delta)/(u_{1o} - u\delta)$; 2) $u/u_{1i}$; 3) $[(\overline{u'v'})_o/u^2_m]\cdot 10^2$; 4) $[(\overline{u'v'})_i/u^2_m]\cdot 10^2$; 5) data of Wygnanski and Fiedler; 6) Childs.

ently due to the effect of the curvature of the streamlines (see [8, 9]), and in the inner zone to the effect of the circulation region.

In [10], which gives the results of an investigation of the friction stresses behind a plane step near the dividing streamline, the growth of the friction stresses along the length of the circulation zone is also noted. Despite the fact that in the inner mixing zone for a given velocity profile the curvature should reduce the friction stresses, they increase as compared with an ordinary mixing layer. From this we may conclude that the effect of the circulation zone is much stronger than that of the curvature.

It should be noted that although, in practice, the velocity profiles in the inner and outer mixing zones are similar, in the turbulence characteristics a considerable deviation from self-similarity is observed. This conclusion is similar to that drawn in [11] in connection with the investigation of a wall jet on a cylindrical surface.

It should also be noted that the data on the fluctuation characteristics obtained by investigating the initial section of a plane jet in a cross flow are consistent with the as-

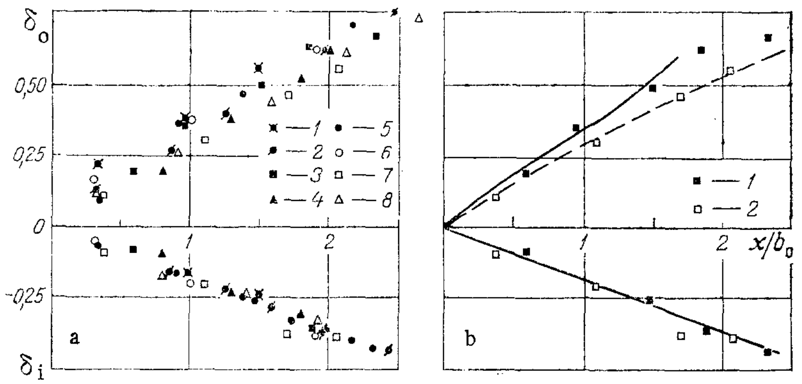


Fig. 4. Conventional widths of upstream and downstream mixing zones. a. 1-5) $H/b_0 = 20$; 6-8) $H/b_0 = \infty$; 1) $u_0/V_\infty = 1.83$; 2) 3.74; 3) 2.91; 4) 3.21; 5) 2.24; 6) 2.34; 7) 2.87; 8) 3.23 [1]. b. 1) $H/b_0 = 20$, $u_0/V_\infty = 2.91$; 2) $H/b_0 = \infty$, $u_0/V_\infty = 2.87$.

sumption made in [2] concerning additional ejection into the outer (upstream) mixing zone of a plane jet propagating in an unconfined flow as compared with the ejection for an ordinary submerged jet.

Since the velocity in the jet core is variable along the radius, the upstream edge of the core and the velocity $u_{1o}$ were determined by means of the profile of the fluctuation velocity $\sqrt{\overline{V'^2}}/V_m$, where $V_m$ is the maximum velocity in the jet core. We first found the inner edge of the core $y_{2i}$ as the ordinate at which the velocity $u_{1i}$ is equal to 0.99 of the maximum. On the fluctuation velocity profile at $y = y_{2i}$ we found the corresponding value of $\sqrt{\overline{V'^2}}/V_m$. Considering that the curvature of the jet has practically no effect on the fluctuation velocity in the core, from the value obtained for $\sqrt{\overline{V'^2}}/V_m$ we determined the ordinate of the outer edge of the core $y_{2o}$ and the corresponding velocity $u_{1o}$.

The conventional widths of the mixing zones were determined as the difference of the ordinates of the lines on which the excess velocity is equal to 0.2 and 0.9 of the excess velocity in the core:

$$u - u_{\delta i} = 0.2\,(u_{1i} - u_{\delta i}),\quad u = u_{\delta i} = 0.9\,(u_{1i} - u_{\delta i})$$

(i = o for the outer mixing zone; i = i for the inner zone).

The boundaries were found from the profiles of the modulus of the velocity vector: since the conventional boundaries of the jet were determined at a sufficient distance from the actual boundaries, in view of the smallness of the angles between the velocity vector and the jet axis in this region it may be assumed that the longitudinal component is approximately equal to the modulus of the velocity vector.

In Fig. 4a we have plotted the width of the mixing zones determined by the above-indicated method for two values of the degree of confinement of the flow and different values of the ratio $u_0/V_\infty$. The white symbols represent the mixing zone widths for a jet in an unconfined flow, the black ones those for a jet in a channel with $H/b_0 = 20$. It should be noted that the abscissas of the points taken from [1] include a correction that takes into account the experimental conditions (difference in the value of the empirical constant $C_1$) introduced by multiplying by the ratio of the constants in these experiments to those in the experiments described in the article in question. Clearly, the width of the inner mixing zone is practically independent of both the quantity $u_0/V_\infty$ and the degree of confinement of the flow. For the width of the upstream mixing zone the scatter of the experimental points is appreciable and it may be concluded that as $u_0/V_\infty$ increases the width of the mixing zone decreases and the expansion of the jet is somewhat greater in a channel than in an unconfined flow, evidently because of the effect of the pressure gradient across the jet, which is considerably greater in the case of a channel.

As noted above, an integral solution of the problem of the entrance length of a plane jet in an unconfined cross flow was obtained in [2] with allowance for additional ejection into the upstream mixing zone. In order to solve the problem of a jet developing in a confined flow, it is possible to employ the same approach as in [2]: for determining the boundaries of the outer and inner zones by the integral method we use the condition of conservation of momentum in the downstream part of the jet, the integral relation for the excess momentum of the upstream part of the jet, and the integral equations for the change of flow rate in the upstream and downstream parts of the jet. In this case, as in [2], the change of flow rate in the upstream part of the jet is taken equal to the sum of two components, one of which is proportional to the excess velocity in the jet core (relative to the velocity at the upstream edge of the jet) and the other to the component of the flow velocity normal to the jet axis. Since in the experiments we observed a certain influence of the degree of confinement of the flow on the width of the upstream mixing zone, we introduced into the second component a correction containing the quantity $b_0/H$ and taking into account the fact that as $b_0/H$ increases so does the ejection into the upstream mixing zone. In this case the system of initial equations has the form

$$\frac{d}{dx}\int\limits_{y_{1i}}^{0}(\rho u^2+p)\,dy=0, \tag{1}$$

$$\frac{d}{dx}\int\limits_{y_{1i}}^{0}\rho u dy=C_1\rho u_{1i}, \tag{2}$$

$$\frac{d}{dx}\int\limits_{0}^{y_{10}}\left[u\,(u-u_\delta)+\frac{p}{\rho}\right]dy=y'_{10}\frac{p_\delta}{\rho}-u'_\delta\int\limits_{0}^{y_{10}}udy, \tag{3}$$

$$\frac{d}{dx}\int\limits_{0}^{y_{10}}udy=C_1\,(u_{10}-u_\delta)+C_2V_\infty\left[1+C_3\left(\frac{b_0}{H}\right)^n\right]\cos\alpha. \tag{4}$$

Here $u_\delta$ and $p_\delta$ are the velocity and pressure at the leading edge of the jet which, generally speaking, considering the small variation of the radius of curvature of the jet on the entrance length, should be determined from the flow past an array of cylinders of radius R in an unconfined potential stream.

It should be noted that for a jet in a channel the boundary conditions change since, as shown in [3], the underpressure behind a jet in a channel is much greater than behind a jet in an unconfined flow, and hence the shape of the displacement body formed by the jet, the radius of curvature, and the velocity at the leading edge of the jet change considerably. These quantities can be found with the aid of [3, 12]. It should also be noted that, according to estimates, on the entrance length the velocities and pressures at the surface of a cylinder forming part of an array and an individual cylinder almost coincide and can be determined using the simple formulas for unconfined potential flow past a cylinder (after first having determined the radius with allowance for the underpressure behind the channel jet).

We note that the pressure p in (1) and (3) is reckoned from the pressure $p_0$ in the reverse flow zone beyond the jet.

Making the experimentally confirmed assumption of similarity of the velocity profiles in the leading and trailing parts of the jet, after manipulation the solution of (1)-(4) gives:

$$\delta_2=-\frac{C_1\int\limits_0^x(u_{1i}/u_0)\,dx+R\,(1-u_{1i}/u_0)}{0.55\,u_{1i}}, \tag{5}$$

$$y_{2i}=-b_{02}+\frac{R}{2}(u_{1i}^2/u_0^2-1)-2\delta_2\frac{u_{1i}^2}{u_0^2}(0.416-0.106\,\delta_2/R), \tag{6}$$

$$\delta_1 = \frac{C_1\left[I_1 - 2\frac{V_\infty}{u_0}R\left(1-\cos\frac{x}{R}\right)\right] + C_2\frac{V_\infty}{u_0}R\sin\frac{x}{R}\left[1 + C_3\left(\frac{b_0}{H}\right)^n\right] + R\left(\frac{u_{10}}{u_0} - e^{-1/R}\right)}{0.55\frac{u_{10}}{u_0} + 0{,}45\frac{u_\delta}{u_0}}, \tag{7}$$

$$\begin{aligned} y_{20} = {} & b_{01}\left[1-(1+\Delta p)\frac{V_\infty^2}{u_0^2}\right] + \frac{R}{2}\left[\frac{u_{10}^2}{u_0^2} + 4\frac{u_\delta}{u_0}\left(e^{-b_{02}/R} - \right.\right. \\ & \left.\left. - \frac{u_{10}}{u_0}\right) - e^{-2/R}\right] - 2\delta_1\left(\frac{u_{10}}{u_0} - \frac{u_\delta}{u_0}\right)\left(0{,}416\frac{u_{10}}{u_0} + 0{,}134\frac{u_\delta}{u_0}\right) - \\ & - 2\frac{\delta_1}{R}\left[\delta_1\left(0{,}309\frac{u_{10}^2}{u_0^2} + 0{,}124\frac{u_{10}u_\delta}{u_0^2} + 0{,}068\frac{u_\delta^2}{u_0^2}\right) + \right. \\ & \left. + \frac{R}{2}\left(\frac{u_{1i}^2}{u_0^2} - \frac{u_{10}^2}{u_0^2}\right) - 0{,}416\,\delta_2\frac{u_{1i}^2}{u_0^2}\right] + 2y_{10}\frac{p_\delta}{\rho u_0^2} - 2I. \end{aligned} \tag{8}$$

Here

$$I = \int_0^x \left\{y_{10}\left(\frac{p_\delta}{\rho u_0^2}\right)' + \frac{u_\delta'}{u_0}\left[R\left(e^{-b_{02}/R} - \frac{u_{10}}{u_0}\right) + \delta_1\left(0{,}55\frac{u_{10}}{u_0} + 0{,}45\frac{u_\delta}{u_0}\right)\right]\right\}dx, \tag{9}$$

$$I_1 = \int_0^x (u_{10}/u_0)\,dx,$$

$b_{01}$ and $b_{02}$ are the widths of the upstream and downstream parts of the jet at the nozzle exit.

In accordance with [1], the velocity in the core is found from the expression

$$\frac{u_1}{u_0} = \exp\left[-(b_{02}+y)/R\right]. \tag{10}$$

The velocity and pressure at the leading edge of the jet can be found from the known relations

$$\frac{u_\delta}{V_\infty} = 2\sin\frac{x}{R}, \quad \frac{2p_\delta}{\rho V_\infty^2} = 1 + \Delta p - u_\delta^2/V_\infty^2. \tag{11}$$

System (5)-(8) is easily solved by successive approximations. In this case the quantities $b_{01}$ and $b_{02}$ can be determined from the condition that in the upstream and downstream parts of the jet the core ends at the same point.

We note that approximate values of the integrals I and $I_1$ (9) can be found from the expressions

$$\begin{aligned} I \simeq a_0 \Big( & a_0C_1x\cos\frac{2x}{R} + 2C_1x\sin\frac{x}{R} - a_0C_1\frac{R}{2}\sin\frac{2x}{R} + \\ & + 2b_{01}\sin\frac{x}{R} + a_0b_{01}\cos\frac{2x}{R} - a_0b_{01} + 2C_1R\cos\frac{x}{R} - 2C_1R\Big), \end{aligned} \tag{12}$$

$$I_1 \simeq (1-1/R)\,x + 1{.}552\,C_1x^2/R, \quad a_0 = V_\infty/u_0, \tag{13}$$

which are obtained if the exponential in the expression for $u_{10}$ is expanded in a series in $(b_{02} + y_{20})/R$, only the first two terms being retained, and if for the width of the upstream mixing zone and its boundaries we use the corresponding expressions for a submerged jet.

In Fig. 4b the results of calculating the width of the mixing zones from relations (5)-(9) for $H/b_0 = \infty$, $u_0/V_\infty = 2.87$ (dashed curve) and $H/b_0 = 20$, $u_0/V_\infty = 2.91$ (continuous curve) are compared with the experimental data. The value of the empirical constant $C_1$ was determined from an experiment on an ordinary submerged jet and was found to be $C_1 = 0.044$. The value of the empirical constant $C_2$ was taken to be the same as in [2] for a jet in an unconfined cross flow: $C_2 = 0.1$. The empirical constants $C_3$ and n, introduced to take into account the degree of confinement of the flow, were determined for $H/b_0 = 20$ and a ratio $u_0/V_\infty = 3.74$ and found to be $C_3 = 1$, $n = 0.25$. It is clear from the figure that the results of the calculations are in perfectly satisfactory agreement with the experimental data. It should be noted that, as distinct from the case of the inner mixing zone, the dependence of the width of the upstream mixing zone on x is not linear. As was to be expected, the rate of growth of the width of the upstream mixing zone increases as the ratio of the jet and flow velocities decreases.

## NOTATION

$\underline{a}_0 = V_\infty/u_0$; $b_0 = b_{01} + b_{02}$, nozzle width; $C_1$, $C_2$, $C_3$, and n, empirical constants; H, depth of the channel; p, static pressure; $p_\delta$, static pressure at the leading edge of the jet; R, radius of curvature of the jet axis; u, longitudinal component of the mean velocity; $u_{1o}$ and $u_{1i}$, velocities at the leading and trailing edges of the jet core; $u_0$, jet velocity at the trailing edge of the nozzle; $V_\infty$, velocity of the cross flow; x and y, Cartesian coordinates tied to the jet axis; $y_{1o}$ and $y_{1i}$, ordinates of the upstream (outer) and downstream (inner) edges of the jet; $y_{2o}$ and $y_{2i}$, ordinates of the upstream and downstream edges of the core.

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